

Chapter (2)

does not meet	$b^2 - 4ac < 0$
two point	$b^2 - 4ac > 0$
two equal	$b^2 - 4ac = 0$

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1. Find the values of k for which the line $y = kx - 3$ does not meet the curve

$y = 2x^2 - 3x + k$ [5]

$$kx - 3 = 2x^2 - 3x + k$$

$$0 = 2x^2 - 3x + k - kx + 3$$

$$2x^2 - 3x + k - kx + 3 = 0$$

$$2x^2 - 3x - kx + k + 3 = 0$$

$$2x^2 - 3x - kx + k + 3 = 0$$

$$a = 2, b = -3 - k, c = k + 3$$

$$b^2 - 4ac < 0$$

$$(-3 - k)^2 - 4(2)(k + 3) < 0$$

$$9 + 6k + k^2 - 8(k + 3) < 0$$

$$9 + 6k + k^2 - 8k - 24 < 0$$

$$k^2 - 2k - 15 < 0$$

$$(k - 5)(k + 3) < 0$$

$$-3 < k < 5$$



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2. Solve the simultaneous equations

$$3x^2 - xy + 2y^2 = 16$$

$$2y - x = 4 \quad [5]$$

$$\rightarrow -x = 4 - 2y$$

$$x = 2y - 4$$

$$3(2y-4)^2 - (2y-4)y + 2y^2 = 16$$

$$3(4y^2 - 16y + 16) - 2y^2 + 4y + 2y^2 - 16 = 0$$

$$12y^2 - 48y + 48 + 4y - 16 = 0$$

$$12y^2 - 44y + 32 = 0$$

$$(3y-8)(y-1) = 0$$

$$y = \frac{8}{3}$$

$$x = 2y - 4$$

$$= \frac{16}{3} - 4$$

$$= \frac{16-12}{3}$$

$$= \frac{4}{3}$$

$$y = 1$$

$$x = 2y - 4$$

$$= 2 - 4$$

$$= -2$$

3. (a) Find the set of values of x for which $4x^2 + 19x - 5 \leq 0$. [3]

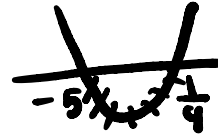
$$x = \frac{1}{4}$$

$$x = -5$$

$$4x^2 + 19x - 5 \leq 0$$

$$(4x - 1)(x + 5) \leq 0$$

$$-5 \leq x \leq \frac{1}{4}$$



- (b) (i) Express $x^2 + 8x - 9$ in the form $(x + a)^2 + b$, where a and b are integers. [2]

$$x^2 + 8x - 9 = (x + a)^2 + b$$

$$= x^2 + 2ax + a^2 + b$$

$$8x = 2ax$$

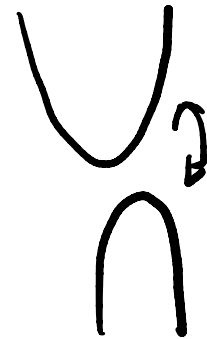
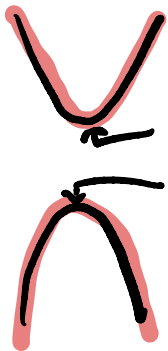
$$a = 4$$

$$a^2 + b = -9$$

$$16 + b = -9$$

$$b = -25$$

$$x^2 + 8x - 9 = (x + 4)^2 - 25$$



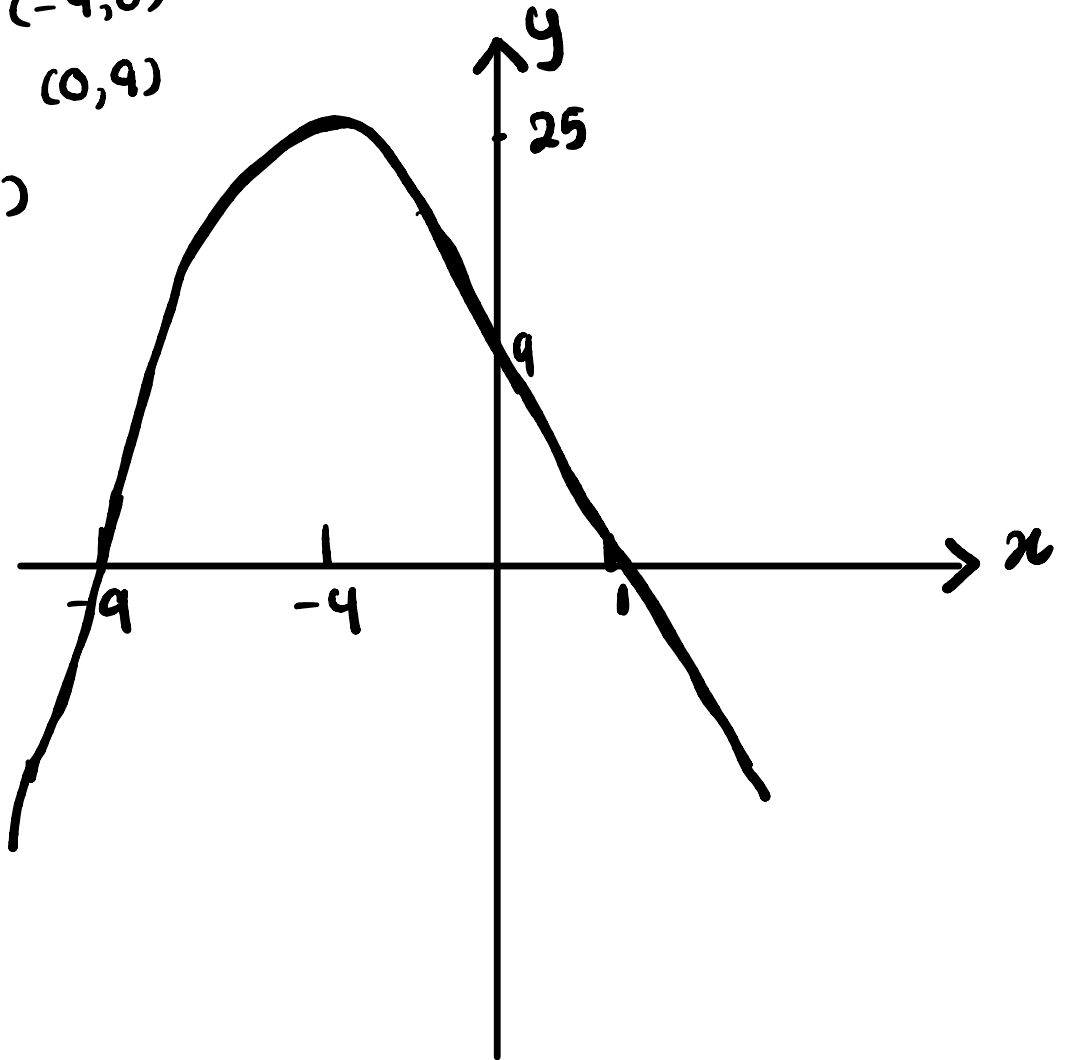
- (ii) Use your answers to part (i) to find the greatest value of $9 - 8x - x^2$ and the value of x at which this occurs. [2]

$$\text{Greatest value} = 25$$

$$\text{value of } x = -4$$

(iii) Sketch the graph of $y = 9 - 8x - x^2$, indicating the coordinates of any point of intersection with the coordinate axes. [2]

$$y=0, 9-8x-x^2=0$$
$$x=1 \quad x=-9$$
$$(1,0) \quad (-9,0)$$
$$x=0, y=9 \quad (0,9)$$
$$(-4,25)$$



4. Find the range of values of k for which the equation $kx^2 + k = 8x - 2xk$ has 2 real distinct roots. [4]

$$kx^2 + k + 2kx - 8x = 0$$

$$kx^2 + 2kx - 8x + k = 0$$

$$a = k, \quad b = 2k - 8, \quad c = k$$

$$b^2 - 4ac > 0$$

$$(2k - 8)^2 - 4(k)(k) > 0$$

$$\cancel{4k^2} - 32k + 64 - \cancel{4k^2} > 0$$

$$-32k + 64 > 0$$

$$-32k > -64$$

$$k < \frac{64}{32}$$

$$k < 2$$

5. A function f is such that $f(x) = x^2 + 6x + 4$ for $x \geq 0$.

(i) Show that $f(x)$ can be written in the form $(x + a)^2 + b$, where a and b are integers. [2]

$$\begin{aligned}x^2 + 6x + 4 &= x^2 + 2ax + a^2 + b \\2a &= 6 & a^2 + b &= 4 \\a &= 3 & a + b &= 4 \\& & b &= -5\end{aligned}$$

$$x^2 + 6x + 4 = (x + 3)^2 - 5$$

\hookrightarrow min
 $(-3, -5)$

(ii) Write down the range of f . [1]

$$y \geq 4$$

(iii) Find f^{-1} and state its domain [3]

$$y = (x + 3)^2 - 5$$

$$x = (y + 3)^2 - 5$$

$$x + 5 = (y + 3)^2$$

$$\sqrt{x + 5} = y + 3$$

$$\sqrt{x + 5} - 3 = y$$

$$f^{-1}(x) = \sqrt{x + 5} - 3, x \geq 4$$

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6. (a) Sketch the graph of $y = |x^2 - 4x - 12|$ showing the coordinates of the point where the graph meets the axes. [3]

$$y = x^2 - 4x - 12$$

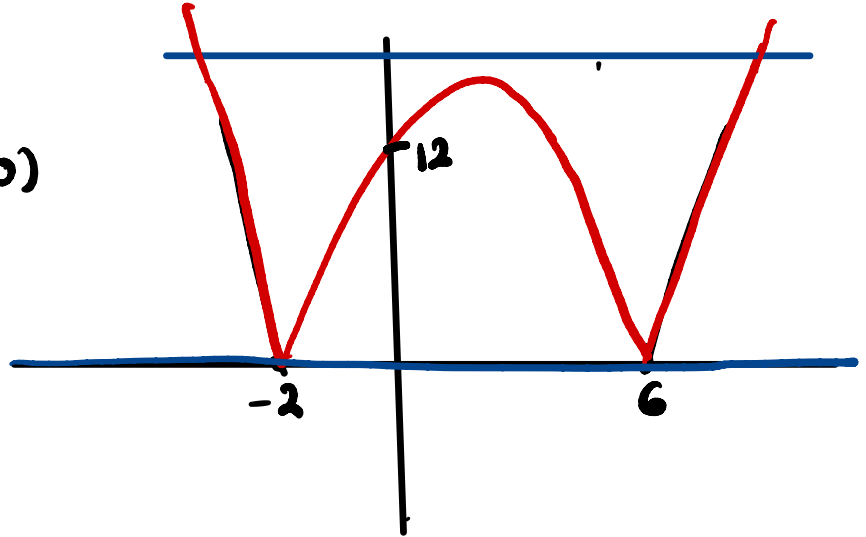
$$x=0, y = -12 \quad (0, -12)$$

$$y=0, x=6, x=-2 \quad (6,0) \quad (-2,0)$$

stationary pt:

$$x = \frac{6+(-2)}{2} = \frac{4}{2} = 2$$

$$\begin{aligned} y &= 4 - 4(2) - 12 \\ &= 4 - 8 - 12 \\ &= -16 \quad (2, -16) \end{aligned}$$



- (b) Find the coordinate of the stationary point on the curve $y = |x^2 - 4x - 12|$. [2]

$$(2, 16)$$

- (c) Find the value of k such that the equation $|x^2 - 4x - 12| = k$ has only 2 solutions. [2]

$$k=0 \quad \text{or} \quad k > 16$$

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7. Given that $f(x) = 3x^2 + 12x + 2$,

a. Find the values of a, b and c such that $f(x) = a(x + b)^2 + c$, [3]

$$3x^2 + 12x + 2 = a(x^2 + 2bx + b^2) + c$$
$$= ax^2 + 2abx + ab^2 + c$$

$$a = 3$$

$$2ab = 12$$

$$6b = 12$$

$$b = 2$$

$$ab^2 + c = 2$$

$$3 \times 4 + c = 2$$

$$c = 2 - 12$$

$$c = -10$$

b. State the minimum value of $f(x)$ and the value of x at which it occurs, [2]

$$x + 2 = 0$$
$$x = -2$$
$$f(x) = 3(x + 2)^2 - 10 \rightarrow (-2, -10)$$

minimum value of $f(x) = -10$
value of $x = -2$

$$3(-2 + 2)^2 - 10$$

c. Solve $f\left(\frac{1}{y}\right) = 0$, giving each answer for y correct to 2 decimal places. [3]

$$3 \times 0 - 10$$
$$0 - 10$$

$$f(x) = 3(x + 2)^2 - 10$$

$$3(x + 2)^2 - 10 = 0$$

$$3(x + 2)^2 = 10$$

$$(x + 2)^2 = \frac{10}{3}$$

$$x + 2 = \sqrt{\frac{10}{3}}$$

$$x = \sqrt{\frac{10}{3}} - 2$$

$$\frac{1}{y} = \sqrt{\frac{10}{3}} - 2$$

$$y = \frac{1}{\sqrt{\frac{10}{3}} - 2} = -5.74$$

$$y = \frac{1}{-\sqrt{\frac{10}{3}} - 2} = -0.26$$

$$\text{or } x + 2 = -\sqrt{\frac{10}{3}}$$

$$x = -\sqrt{\frac{10}{3}} - 2$$

$$\frac{1}{y} = -\sqrt{\frac{10}{3}} - 2$$